**Homework of General Physics II set 1**

1. **(Hecht’s 9.40 with some variation)Given that the mirrors of a Fabry-Perot interferometer have amplitude reflection coefficient of *r*=0.8944, find:**
   1. **The coefficient of finesse:**
   2. **The finesse:**
   3. **The contrast factor defined by: C=(It/I0)max/(It/I0)min**

Answer:

The contrast factor:

* 1. **For 500 nm light, if the cavity length of the F-P is 1 cm, calculate the spectral FWHM (full width at half maximum) in wavelength and in frequency; and its free spectral range (in frequency) for this cavity; and its minimum resolvable wavelength difference around 500nm.**

Answer:

Free spectral range in frequency for this cavity:

Minimum resolvable wavelength difference around 500nm:

* 1. **If I used the above F-P as an etalon, the input light is around 500nm with a frequency distribution width about 10GHz, please roughly estimate how much energy will pass through the etalon (Roughly estimate means you do not need to use Airy function, the exact solution of the output I, to do the calculation. You can not do this anyway because I do not give you the exact form of the frequency distribution of the input. Just estimate using the bandwidth)**

Answer:

Only one frequency of the input light can pass through.

“Just compare the output and input’s line width; you will have a rough idea of the energy percentage.”

2.

1)For a given F-P, we have the maximum output (constructive interference) condition:



But for the same h, it may also correspond to other wavelength (or frequency) with different m. The next higher frequency (or smaller wavelength) would be:



This frequency will also satisfy the constructive interference and will have maximum output at h, i.e. this F-P cannot tell them apart.

The difference in frequency is clearly c/2nh, which is exactly the free spectral range for a given F-P with cavity length h.

(Actually as long as frequency components differs by an integer multiplies c/2nh, they cannot be distinguished by the given F-P)

2) One simple way is this: Since the c/2nh depends on the cavity length h, the easiest way to tell the two frequencies apart is by using a different F-P, preferably with smaller h, say d<h; than the free-spectral range in the new F-P would be c/2nd, the  above would be not differ by free-spectral-range in the new F-P, and will appear as two distinguishable peaks at the output.

1. **Consider the interference of the Michelson interferometer as arising from two beams of equal energy flux density, so the interference pattern can be described by relation 4.1 (Pg170, Zhao’s book).**   
   **. In analogy to what we have treated in the F-P case, derive the expression for the FWHM in phase domain, and what is its finesse?**

Answer:

FWHM in phase domain:

Or just use the maximum happens at, half of maximum happens at, so FWHM isπ.

Spacing between two adjacent peaks:

4. This problem shows you how to use Fabry-Perot spectrometer (scanning h) to know the frequency differences of light:

Consider a Fabry-Perot scanning spectrometer, with n=1, h=1.5cm; Finesse=100. The h can be increased by applying voltage to piezo material. All light input are at normal incident angle to the F-P mirror. For a reference light whose wavelength is known as , another light with unknown wavelength (or frequency ) which is very close to . As we scan the F-P (change its h), on the oscilloscope (the horizontal axis is scanning time which in turn is proportional to the piezo-voltage, the voltage in turn is proportional to the length of F-P cavity, so we can say the horizontal is proportional to h; the vertical axis is intensity of light at the output) we observe the following picture:





The reason that you have multiple-sets is that they correspond to different interference order, say the 1st set corresponds to order m; the 2nd will be m+1 (if you scan h further, you will see more sets). By measuring the spacing of the peaks, we can determine the wavelength difference between , and thus if we know , then  is known too. Here provides with parameters given above, and  is the spacing between peaks of  within same set (same order of m);  is the spacing between the peaks of adjacent order, and , what is the wavelength of  and frequency ?

Answer:

The relation between wavelength of maximum output and cavity length is:

, (1) m is the order of interference.

Different wavelength will appear at different h, so that:





So if we know the and m we know wavelength difference.

To know , we use ,

 is the spacing between m+1 and m order, then from relation (1):



With n=1, :



Then: 

For the order number m, also using (1), with  h=1.5cm



Then: . 

It is a small wavelength (or frequency difference), then I can just differential relation for frequency difference:





Above is analysis from wavelength, a quicker way to get frequency difference (and thus wavelength difference) is using free spectral range: (This is another method which will give you same answer)

For the given F-P:

Free spectral range 

The spacing between peaks of adjacent order corresponds to this  (imagine  their frequency difference is one free spectral range, then the mth order of  will overlap with the m+1 order of , this is the conclusion of problem 2), then:



This is same as above.

1. **In the discussion of multi-beam interference and Fabry-Perot, we assume the two reflecting surfaces are lossless, i.e. the total energy are either reflected or transmitted. In this case, the transmitted light can reach 100% Io. However in real application for high finesse F-P, transmission seldom exceeds 20%, higher the finesse, lower the transmission. This is of cause due to the loss of light when interacting with the surface (such as absorption, scattering and diffraction etc). For a cavity formed by two parallel high reflecting surfaces, with R=r2=0.99 (here R, r are the reflection for the ideal lossless case), but due to some reason (such as dirty mirror) a loss occurs. For each time the light hits the surface, 4% of its energy is lost. With such cavity, what is the maximum transmission of light?**

**[Hint: The important thing is what is the energy or the field distributions when light interacts with the surface each time. For the lossless surface, we have R+T=1, where R=r2, T=t2 if n is same on both sides of the thin reflecting film. (If n’s are different, it will complicate the estimation a little but will not affect the basic idea). R, r, T, t are for lossless case. We can write the reflecting and transmitting field and work out the problem as in the class. For the surface where 4% energy is lost every time the light hits it, it is R”+T”+A=1. A is the loss (0.04 here), R” and T” are reflected and transmitted. We make an assumption that the loss is same for reflection and transmission, then R”=*a*2r2, T”=*a*2t2. You can find out the *a* and write out the reflected and transmitted field at each interaction between light and surface, the rest would be similar to the derivation I gave in the class; You may also read Hecht book where he discussed this issue in the F-P part, but I encourage you derive the result yourself]**

Answer:

1. **A tungsten-filament lamp having a straight filament that is 0.1 mm in diameter is used as a source for a Young’s interference experiment, the wavelength is ~590 nm. A double-slit aperture with 1mm between the slits has to be placed at least to what distance in order to observe the interference fringes? If we require the degree of coherence (the contrast) is more than 90%, what is the distance? Should the slits be oriented parallel to the lamp filament?**

Answer:

Critical condition to observe the interference fringes:

∴The double-slit aperture between the slits has to be placed more than 0.169m away.

The contrast:

This equation can be solved graphically or use expansion:

The slits should be oriented parallel to the lamp filament.

Because the spatial coherence is caused by the size of incoherence source TRANSVERSE (perpendicular) to the slits. (That is why spatial coherence is also called transverse coherence.) So we need to reduce the transverse size of the source witch respect to the slit’s arrangement.

1. **Using Michelson interferometer to observe an equal inclination fringes. The light source is a broad band thermal source, with the light centers about 500 nm, with a width of 100 nm. Using such light sources, how many** **interference fringes can you observe?**

Answer:

The OPL length difference between each fringe is justλ,the Nth fringe will have OPL difference= Nλ, it has to be less than the maximum OPL difference, so:

8. Given a Femto-second (10-15 s) laser pulse, (i.e. the duration of pulse in time, or temporal width) estimate its spectral width, i.e. the width in frequency domain. Given the central wavelength of the spectral distribution is 500 nm, what is the spectral width in wavelength (i.e. the approximate longest and shortest wavelengths such pulse contains)? Is it able to excite Sodium transition around 580 nm?

Answer: 

For pulse with , its spectral width in frequency is:



The central wavelength is 500 nm, its frequency is , then the end frequencies of the spectrum will be:

, 



So the femto-second pulse can be thought as superposition of light from UV to IR, and 580nm is within its spectral wavelength range and it can excite sodium transion.

(Because of big wavelength or frequency difference, I do not use differential relation in the estimate of wavelength)